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Optimum Leakage Attacks on Combined Area-Terminal Defense Systems

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Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

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FOR THE COMMANDER

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OPTIMUM LEAKAGE ATTACKS ON COMBINED AREA-TERMINAL DEFENSE SYSTEMS

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Group 32

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ABSTRACT

This report presents a simple methodology for determining the optimum attack on non-uniform valued targets defended by a layered defense consisting of an area defense of all targets and a terminal defense of higher value targets. Only leakage attacks (as opposed to interceptor exhaustion attacks) are considered. Depending on the leakage of the defense layers and the number of targets with terminal defense, the optimum attack emphasizes either high value targets attempting to leak through both layers, or else smaller targets having only area defense. Simple equations governing the attack strategy and the expected damage are derived and sample numerical results are presented.

Introduction

Recently there has been renewed interest in defensive systems to protect national value (population, industry, etc.) as opposed to strategic forces (missiles, bombers, etc.). Systems to protect against ballistic missiles, cruise missiles and bombers have been considered; typically these consist of an area defense layer capable of defending any target and a terminal defense layer capable of defending a relatively small number of high value targets. An attacker trying to penetrate such a layered defense has the option of shooting at the heavily defended, high value targets in the hope of destroying a lot of value should the attack penetrate, or shooting at the more lightly defended, lower value targets for which the defense penetration is easier. This report will present a simple methodology for determining the optimum attack strategy. Only leakage attacks are discussed here; the analysis of interceptor exhaustion attacks is much more complicated.

Target Structure

The defended target set consists of a set of aimpoints of varying values. The value is taken as the damage done by an attacking nuclear weapon detonation at the given aimpoint. This is the value contained within a lethal radius of the aimpoint; the lethal radius is a function of weapon yield and target hardness. For simplicity, the value may be thought of as

population although other measures of value are often used. The defended aimpoints may be rank ordered according to their value. A common value distribution model is Zipf's law and, for ease of computation, it is this model which will be assumed here. Zipf's law states that the value of the aimpoint of rank R is 1/R relative to the value of the highest value aimpoint.

$$V(R) = \frac{1}{R} \tag{1}$$

Fig. 1 illustrates that Zipf's law is a good approximation for the population of urbanized areas; this would correspond to aimpoint value for very high yield weapons. The claim is made, without proof, that a similar ranking of aimpoints for smaller lethal radii could also be approximated by the 1/R dependence. For example, it is expected that the highest value aimpoint (probably lower Manhattan) would have twice the value of the second must valuable aimpoint (probably downtown Chicago), etc.

<u>Defense Model</u>

The functioning of the defense is illustrated in Fig. 2 The area defense layer can intercept missiles attacking any target with leakage L_A; i.e., the probability of each missile penetrating the area defense layer is L_A. The terminal defense layer defends the R_T highest value targets with leakage L_T. Thus the probabilty of each missile penetrating the defense at a high value target is $L_A L_T$ while the penetration probability at a low value target is L_A.

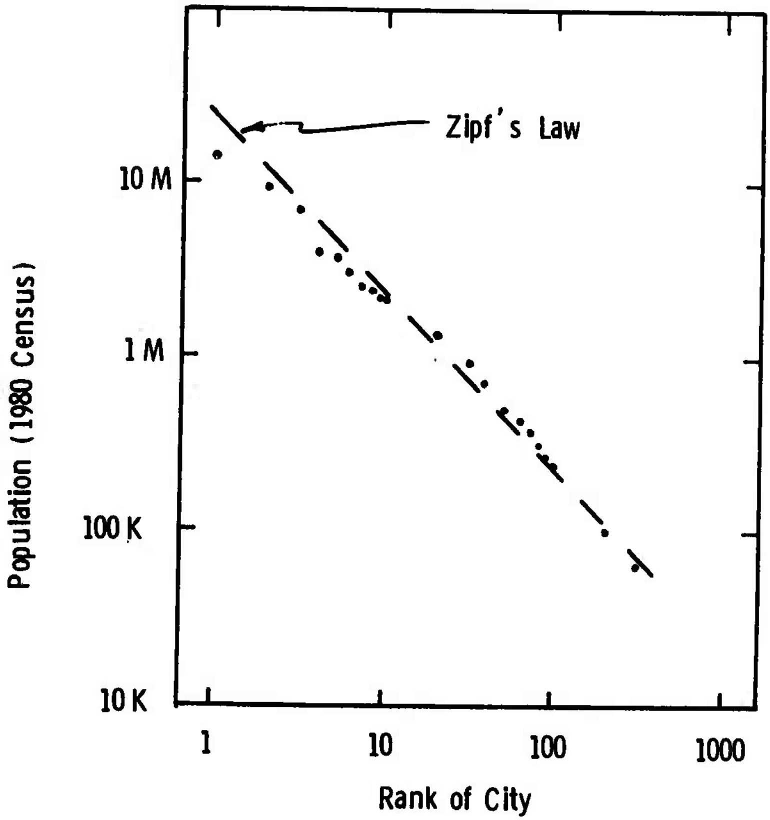

Fig. 1. Population of urbanized areas.

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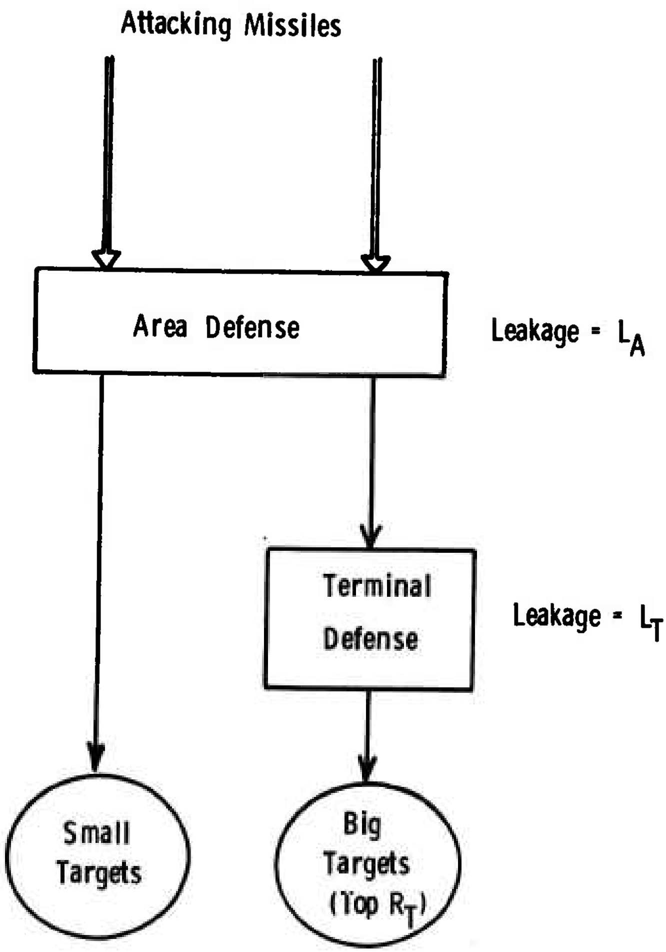

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Fig. 2. Defense model.

Each layer of defense is assumed to have enough interceptors to shoot at each attacking missile, thus exhaustion attacks are not effective. In some of the numerical examples, the number of interceptors required for this to be the case will be indicated.

Attack Optimization

An attack is completely characterized by specifying the number of missiles sent to each target, n(R). This function is chosen to maximize the expected damage subject to a constraint on the total attack size. The total damage is the sum of the expected damage at each target which is calculated below. The probability that a target survives an attack of n missiles is *

$$P_s = e^{-Ln} \qquad \begin{cases} L = L_A L_T & \text{for } R \leq R_T \\ L = L_A & \text{for } R > R_T \end{cases} \tag{2}$$

The expected damage done by the n^{th} missile assigned to the target is the target value multiplied by (-dP/dn) or

$$\text{Incremental Damage} \equiv d = \frac{L}{R} \; e^{-Ln} \tag{3}$$

The term L/R is the expected damage caused by the first missile attacking the target of rank R. The factor e^{-Ln} represents the

*More precisely, $(1-L^*)^n$. The two expressions are equivalent if $L=-\ln(1-L^*)$. For $L<0.1$, the two values of leakage are essentially equal. The version in Eq (2) is used for ease of calculation.

probability that the target has not already been destroyed.

The overall allocation strategy is such that all targets attacked are attacked to the same level of incremental damage. To see that this is the case, consider a counter example where target i is attacked to a level corresponding to incremental damage d_i while target j is attacked to incremental damage $d_j > d_i$. In this case, a superior result (for the attacker) could be obtained by removing one missile attacking target i and using it to attack target j. The additional damage resulting would be $d_j - d_i$ which is positive. Thus this attack assumed cannot be optimal. For the optimal attack, $d_i = d_j$ for all i,j.

Solving Eq (3) for n(R) gives

$$n(R) = -\frac{1}{L} \ln \left(\frac{Rd}{L}\right) \quad \text{for } R < L/d \qquad (4)$$

Given a value of d, Eq (4) determines how the missiles are allocated among the targets. Fig. 3 illustrates sample allocations for various values of d. The total number of missiles used is the summation of n(R) over all targets. For simplicity, the summations will be replaced by integrals. It is convenient to consider terminally defended and non-terminally defended targets separately.

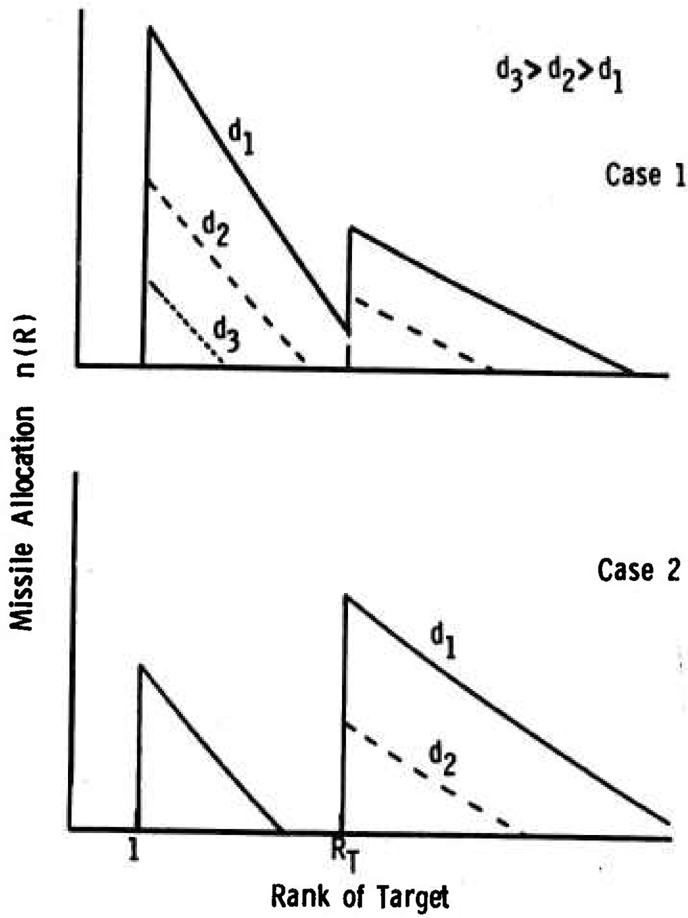

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Fig. 3. Sample allocations.

1) Terminally defended targets

$$\text{Attack size} = T_1 = \int_1^{R^*} n(R)\, dR$$

where

$$R^* = \begin{cases} 1 & \text{for} \quad d > L_A L_T \\ L_A L_T / d & \text{for} \quad L_A L_T > d > L_A L_T / R_T \\ R_T & \text{for} \quad L_A L_T / R_T > d \end{cases}$$

$$\text{or} \quad T_1 = \begin{cases} 0 \\ \dfrac{1}{d} + \dfrac{1}{L_A L_T} \ln\left(\dfrac{d}{L_A L_T}\right) - \dfrac{1}{L_A L_T} \\ \dfrac{1}{L_A L_T} \ln\left(\dfrac{d}{L_A L_T}\right) + \dfrac{R_T - 1}{L_A L_T} - \dfrac{R_T}{L_A L_T} \ln\left(\dfrac{R_T d}{L_A L_T}\right) \end{cases} \quad (5)$$

2) Non terminally defended targets

$$\text{Attack Size} = T_2 = \int_{R_T}^{R^{**}} n(R) dR$$

where

$$R^{**} = \begin{cases} R_T & \text{for} \quad d > L_A / R_T \\ L_A / d & \text{for} \quad L_A / R_T > d \end{cases}$$

$$\text{or} \quad T_2 = \begin{cases} 0 \\ \dfrac{1}{d} + \dfrac{R_T}{L_A} \ln\left(\dfrac{dR_T}{L_A}\right) - \dfrac{R_T}{L_A} \end{cases} \quad (6)$$

The total threat level is $T = T_1 + T_2$. By varying d parametrically, it is possible to determine the allocation n(R) as a function of the threat size T.

Expected Damage

Given $n(R)$ it is possible to calcuate the expected damage at target R. By summing this over R, the total expected damage is obtained. This will also be calculated treating d as a parameter.

The expected damage at target R is the value of that target times the kill probability which is $(1-P_s)$. From Eqs (1) and (2), D(R) is given by

$$D(R) = \frac{1}{R}\,(1-e^{-Ln}) \tag{7}$$

Combining this with Eq (3) yields

$$D(R) = \frac{1}{R} - \frac{d}{L} \qquad \text{for } R < \frac{L}{d} \tag{8}$$

$$\text{where } L = L_A L_T \text{ for } R \leq R_T$$

$$L = L_A \quad \text{for } R > R_T$$

In Eq (8), the term $(1/R)$ represents the total value of the target. It is seen that every target is attacked until only a value (d/L) remains. Those targets with total value less than this level are not attacked at all.

The total damage done by the attack is approximated by the integral of D(R). This is again broken up into the damage on terminally defended targets, D_1, and that on non-terminally defended targets, D_2.

$$D_1 = \int_{1}^{R^*} D(R)dR$$

$$= \begin{cases} 0 & d > L_A L_T \\ \ln\left(\frac{L_A L_T}{d}\right) - 1 + \frac{d}{L_A L_T} & L_A L_T > d > L_A L_T/R_T \\ \ln(R_T) - (R_T - 1)d/L_A L_T & L_A L_T/R_T > d \end{cases} \quad (9)$$

and

$$D_2 = \int_{R_T}^{R^{**}} D(R)dR$$

$$= \begin{cases} 0 & d > L_A/R_T \\ \ln\left(\frac{L_A}{dR_T}\right) - 1 + \frac{dR_T}{L_A} & L_A/R_T > d \end{cases} \quad (10)$$

The total damage is $D=D_1+D_2$. The unit of damage is the total value of the highest value target (aimpoint).

Results

From Eqs (4) - (10), it is possible to determine the optimum allocation n(R), the attack size, T, and its breakdown into T_1 and T_2, the damage distribution, D(R), and the total damage, D, as functions of the incremental damage parameter, d. It is of particular interest to determine the relative targeting to terminally defended and non-terminally defended targets as a function of attack size and relative leakage values. Figs. 4-6 show T_1/T, T_2/T and D as functions of T for a variety of defense models. Several interesting results may be seen. If $L_T R_T > 1$, the attack tends to concentrate on the high value, terminally defended targets while, if $L_T R_T < 1$, the attack concentrates on non-terminally defended targets. The total damage is a monotonic increasing, concave down function of threat size; this is a characteristic of leakage attacks. For the parameters chosen, only a few units of damage result and this level is relatively insensitive to how the leakage is apportioned between area and terminal defense and how many targets are terminally defended.

A final numerical result is the weapon allocation, n(R), for a particular set of defense parameters. Because of significant quantization effects, the allocation to target R is taken to be a corrected version of n(R)

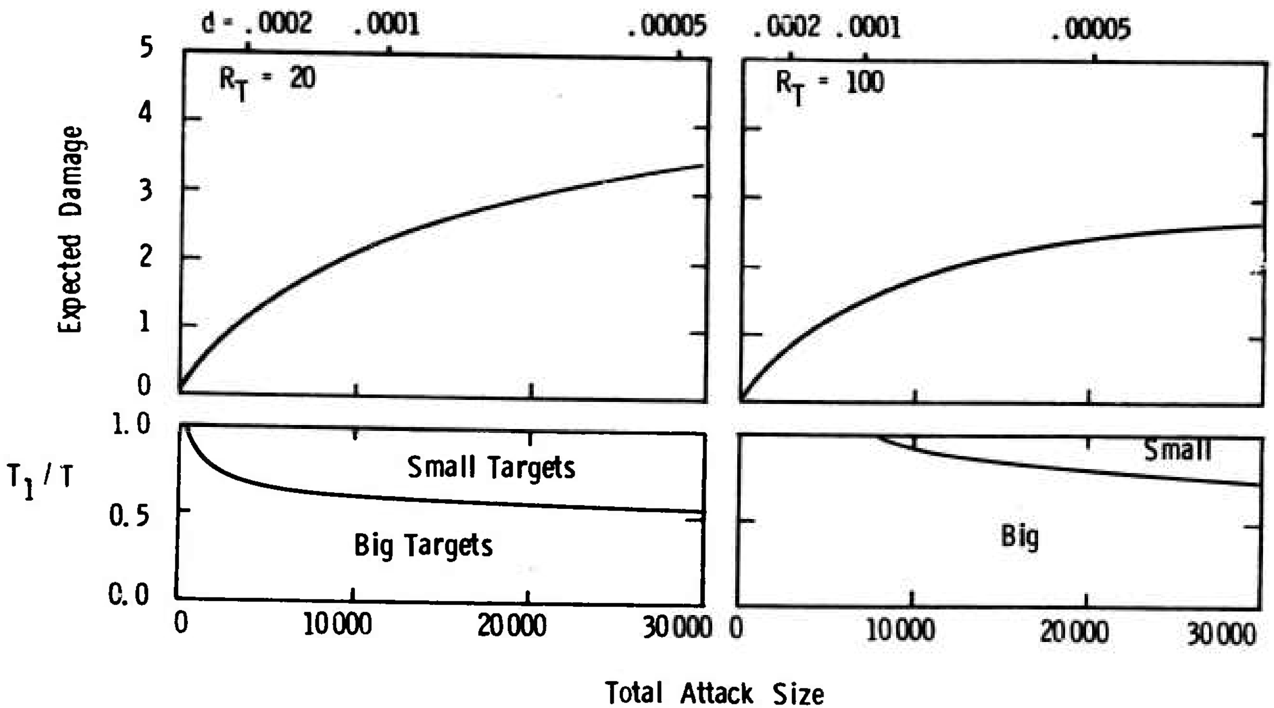

Fig. 4. Numerical results - L_A = .01, L_T = .1.

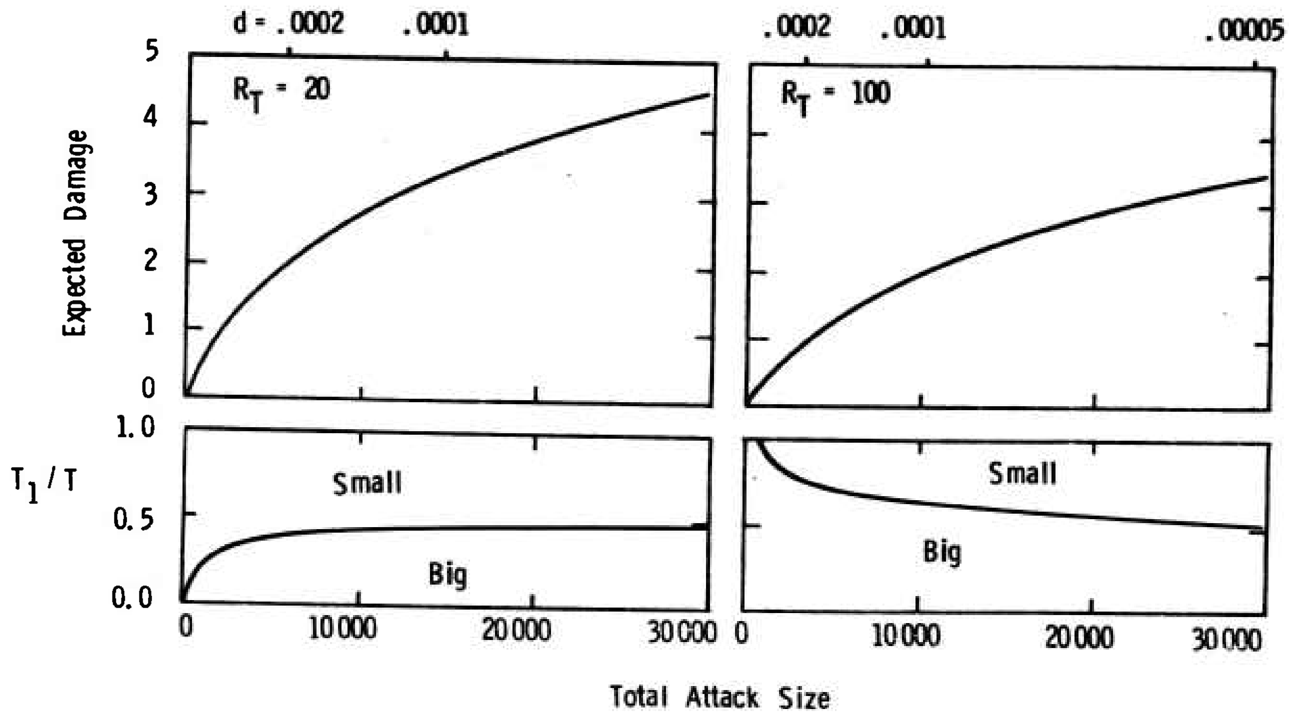

Fig. 5. Numerical results - L_A = .04, L_T = .025.

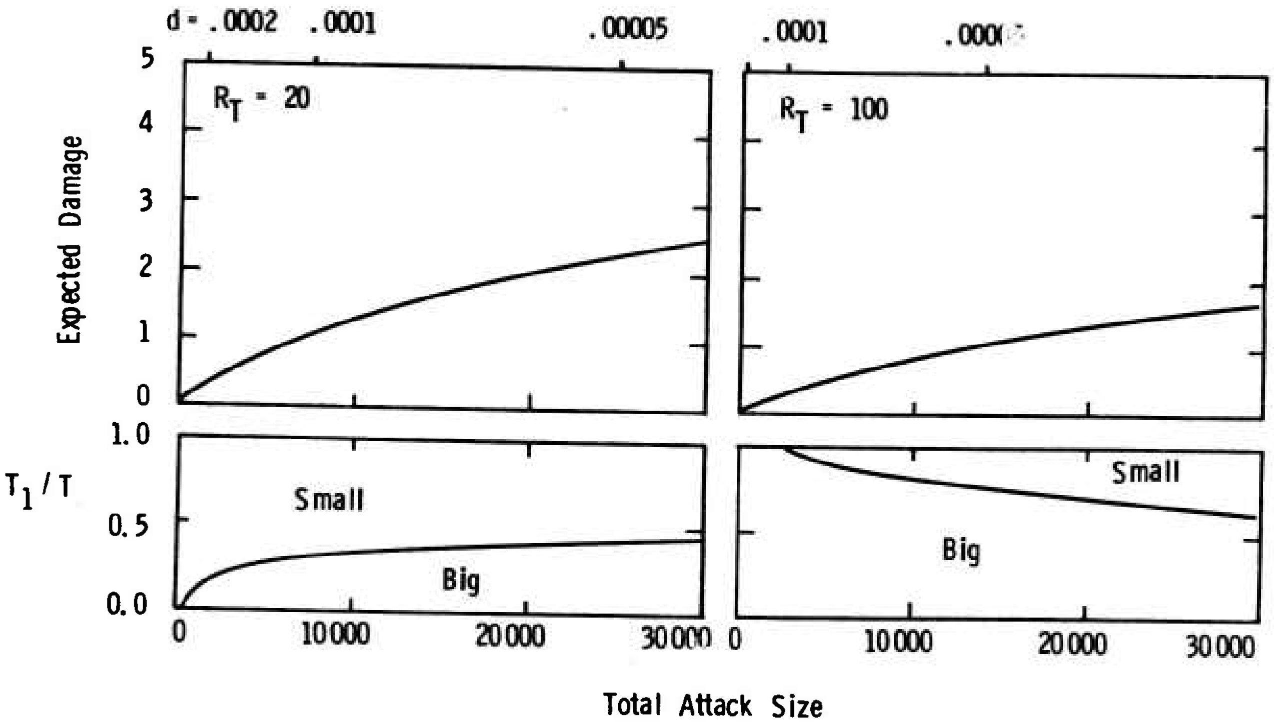

Fig. 6. Numerical results - L_A = .01, L_T = .025.

$$N(R) = \int_{R}^{R+1} n(R)dR \qquad \text{if } N(R) > 0$$

$$= n(R+1) + \frac{1}{L}\left[1 - R \ln\left(\frac{R+1}{R}\right)\right] \tag{11}$$

Tables 1 and 2 show two sample allocations and the resulting damage distributions. The expected numbers of missiles penetrating the area defense are also shown; these are good lower bounds on the number of terminal interceptors required to discourage an exhaustion attack. Of course, an appropriate number of interceptors must also be deployed even at those targets not attacked in Table 1 and 2.

Summary

The methodology presented here permits simple determination of missile allocation for optimum leakage attacks on a combined area/terminal defense and for the expected damage resulting from this attack. Sensitivity to area and terminal leakage and to the number of targets with terminal defense can be easily determined. Caution must be used in extending these results to exhaustion attacks which are much more difficult to analyze.

TABLE 1

Sample Allocation

$L_A=0.1$ $L_T=.1$ $R_T=20$ $d=.00022 (T \approx 3000)$

Rank of Target	N(R)	Kill Probability	# Terminal Int.
1	1128	.676	12
2	605	.454	7
3	265	.233	3
4	12	.012	1
5	0	---	--
...	....	....	...
21	75	.527	
22	70	.505	
23	66	.483	
24	62	.461	
25	58	.439	
26	54	.417	
27	50	.395	
28	47	.373	
29	43	.351	
30	40	.329	
31	37	.307	
32	34	.285	
33	31	.263	
34	28	.241	
35	25	.219	
36	22	.197	
37	19	.175	
38	17	.153	
39	14	.131	
40	12	.109	
41	9	.087	
42	7	.065	
43	4	.043	
44	2	.021	
45	0	----	

TABLE 2

Sample Allocation

$L_A=0.1$ $L_T=.025$ $R_T=100$ $d=.00003(T \approx 30000)$

Rank of Target	N(R)	Kill Probability	# Terminal Int.
1	6936	.823	70
2	4843	.702	49
3	3484	.581	35
4	2473	.461	25
5	1668	.341	17
6	998	.221	10
7	424	.101	5
8	0	----	---
...	...	...	
101	119	.696	
...	...	...	
150	80	.548	
...	...	...	
200	51	.399	
...	...	...	
250	29	.248	
...	...	...	
300	10	.098	
...	...	...	
330	1	.009	
331	1	.009	
332	0	----	

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